CSC 310 - Programming Languages

Context Free Grammars

Languages and Automata

- Formal languages are important in computer science, especially in programming languages.
- Regular languages are the weakest formal languages that are widely used
- We also need to study context-free languages

Limitations of Regular Languages

- Intuition: A finite automaton that runs long enough must repeat states
- A finite automaton cannot remember the number of times it has visited a particular state
- A finite automaton has finite memory, so:
  - it can only store which state it is currently in, and
  - cannot count, except up to a finite limit.
- Example, the language of balanced parentheses is not regular: $\{(^i)^i \mid i \geq 0\}$

The Role of the Parser

- The parsing phase of a compiler can be thought of as a function:
  - Input: sequence of tokens from the lexer
  - Output: parse tree of the program
- Not all sequences of tokens are programs, so a parser must distinguish between valid and invalid sequences of tokens
- So, we need
  - a language for describing valid sequences of tokens, and
  - a method for distinguishing valid from invalid sequences of tokens.

Context-Free Grammars

- Many programming language constructs have a recursive structure
- Example, a statement is of the form:
  - if condition then statement else statement, or
  - while condition do statement, or
  - ...
- Context-free grammars (CFGs) are a natural notation for this recursive structure

Context-Free Grammars

- A context-free grammar consists of
  - A set of terminals T
  - A set of non-terminals N
  - A non-terminal start symbol S
  - A set of productions
- Assuming that $X \in N$, productions are of the form
  - $X \rightarrow \epsilon$, or
  - $X \rightarrow Y_1 Y_2 \ldots Y_n$ where $Y_i \in N \cup T$

Notational Conventions

- In these lecture notes
  - Non-terminals are written in uppercase
  - Terminals are written in lowercase
  - The start symbol is the left-hand side of the first production

CFG Example

- A fragment of a simple language

$$
\begin{aligned}
STMT &\rightarrow \textit{if } COND \textit{ then } STMT \textit{ else } STMT \\
STMT &\rightarrow \textit{while } COND \textit{ do } STMT \\
STMT &\rightarrow \ \textit{id} \ = \ \textit{int}
\end{aligned}
$$

- Notational abbreviation

$$
\begin{aligned}
STMT \rightarrow\ & \textit{if } COND \textit{ then } STMT \textit{ else } STMT \\
\mid\ & \textit{while } COND \textit{ do } STMT \\
\mid\ & \textit{id} \ = \ \textit{int}
\end{aligned}
$$

CFG Example

- Classic CFG example: simple arithmetic expressions

$$
\begin{aligned}
E \rightarrow\ & E * E \\
\mid\ & E + E \\
\mid\ & (E) \\
\mid\ & id
\end{aligned}
$$

The Language of a CFG

- Productions can be read as replacement rules
- $X \to Y_1 \dots Y_n$ means that X can be replaced by $Y_1 \dots Y_n$
- $X \to \epsilon$ means that X can be erased (replaced with the empty string)

The Language of a CFG: Key Idea

1. Begin with a string consisting of the start symbol S
2. Replace any non-terminal X in the string by a right-hand side of some production $X \to Y_1 \ldots Y_n$
3. Repeat step 2 until there are no non-terminals in the string

The Language of a CFG

- Let G be a context-free grammar with start symbol S. Then the language of G ($L(G)$) is:

  $$\{a_1 \ldots a_n \mid S \xrightarrow{*} a_1 \ldots a_n \wedge every\ a_i \in T\}$$

  where

  $$X_1 \ldots X_n \xrightarrow{*} Y_1 \ldots Y_m$$

  denotes

  $$X_1 \ldots X_n \rightarrow \ldots \rightarrow Y_1 \ldots Y_m$$

Terminals

- A terminal has no rules for replacing it, hence the name terminal
- Once a terminal is generated, it is permanent
- Terminals ought to be the tokens of the language

Parentheses Example

- Strings of balanced parentheses $\{(^i)^i \mid i \geq 0\}$
- Grammar

$$\begin{aligned} S &\rightarrow (S) \\ &\mid \epsilon \end{aligned}$$

Example

- A fragment of a simple language

$$
\begin{aligned}
STMT &\rightarrow if\ \ COND\ then\ STMT\ else\ STMT \\
&\mid while\ COND\ do\ STMT \\
&\mid id\ = \ int \\
COND &\rightarrow (id == id) \\
&\mid (id! = id)
\end{aligned}
$$

Example Continued

- Some elements of the language
  - `id = int`
  - `if (id == id) then id = int else id = int`
  - `while (id != id) do id = int`
  - `while (id == id) do while (id != id) do id = int`

Arithmetic Example

- Simple arithmetic expressions:

$$E \rightarrow E + E \mid E * E \mid (E) \mid id$$

- Some elements of the language
  - id
  - (id)
  - (id) * id
  - id $+$ id

Notes

- The idea of a CFG is a big step
- But,
  - Membership in a language is boolean; we also need the parse tree of the input
  - Must handle errors gracefully
  - Need an implementation of CFGs
- Form of the grammar is important
  - Many grammars generate the same language
  - Parsing tools are sensitive to the grammar

Derivations and Parse Trees

- A derivation is a sequence of productions

$$S \rightarrow \ldots \rightarrow \ldots \rightarrow \ldots$$

- A derivation can be depicted as a tree
  - The start symbol is the tree's root
  - For a production $X \rightarrow Y_1 \ldots Y_n$ add children $Y_1 \ldots Y_n$ to node X

Derivation Example

- Simple arithmetic expressions:

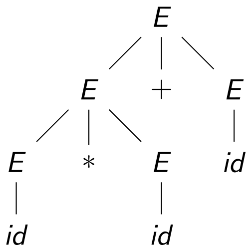
$$E \to E + E \mid E * E \mid (E) \mid id$$

- String

$$id * id + id$$

Derivation Example

$$
\begin{aligned}
& E \\
\rightarrow & E + E \\
\rightarrow & E * E + E \\
\rightarrow & id * E + E \\
\rightarrow & id * id + E \\
\rightarrow & id * id + id
\end{aligned}
$$



Notes on Derivations

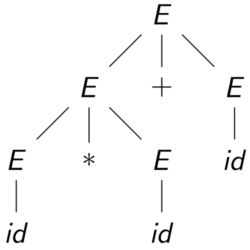
- A parse tree has:
  - terminals at the leaves, and
  - non-terminals at the interior nodes
- An in-order traversal of the leaves is the original input
- The parse tree shows the association of the operations, the input string does not

Left-most and Right-most Derivations

- The previous example was a left-most derivation
  - At each step, replace the left-most non-terminal
- There is an equivalent notion of a right-most derivation
  - At each step, replace the right-most non-terminal

Right-most Derivation Example

$$
\begin{aligned}
& E \\
\to & E + E \\
\to & E + id \\
\to & E * E + id \\
\to & E * id + id \\
\to & id * id + id
\end{aligned}
$$



Derivations and Parse Trees

- Note that right-most and left-most derivations have the same parse tree
- The difference is the order in which branches are added

Summary of Derivations

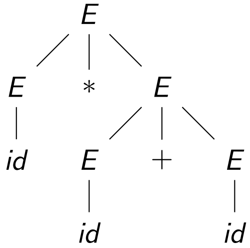
- We are not only interested in whether $S \in L(G)$, we also need a parse tree for S
- A derivation defines a parse tree, but one parse tree may have many derivations
- Left-most and right-most derivations are important in the parser implementation

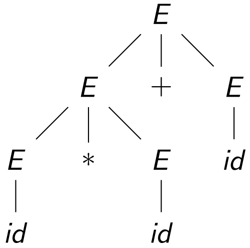
Ambiguity

- Grammar

$$E \rightarrow E + E \mid E * E \mid (E) \mid id$$

- The string $id * id + id$ has two parse trees:





Ambiguity

- A grammar is ambiguous if it has more than one parse tree for some string
- Ambiguity leaves the meaning of some programs ill-defined
- Ambiguity is common in programming languages

Dealing with Ambiguity

- There are several ways to handle ambiguity
- The most direct method is to rewrite the grammar unambiguously
- Example: enforcing precedence in the previous grammar

$$
\begin{aligned}
E &\rightarrow T + E \\
&\mid T \\
T &\rightarrow id * T \\
&\mid id \\
&\mid (E)
\end{aligned}
$$

Ambiguity: The Dangling Else

- Consider the following grammar

$$
\begin{aligned}
S \rightarrow\ & \mathit{if}\ C\ \mathit{then}\ S \\
\mid\ & \mathit{if}\ C\ \mathit{then}\ S\ \mathit{else}\ S \\
\mid\ & \mathit{OTHER}
\end{aligned}
$$

- This grammar is ambiguous: the expression "$\mathit{if}\ C_1\ \mathit{then}\ \mathit{if}\ C_2\ \mathit{then}\ S_3\ \mathit{else}\ S_4$" has two parse trees

The Dangling Else: a Fix

- We want "else" to match the closest unmatched "then"
- We can describe this in the grammar

$$
\begin{aligned}
S &\rightarrow MIF \\
&\mid UIF \\
MIF &\rightarrow if\ \ C\ then\ MIF\ else\ MIF \\
&\mid OTHER \\
UIF &\rightarrow if\ \ C\ then\ S \\
&\mid if\ \ C\ then\ MIF\ else\ UIF
\end{aligned}
$$

Ambiguity

- No general techniques for handling ambiguity
- Impossible to automatically convert an ambiguous grammar to an unambiguous one
- Used with care, ambiguity can simplify the grammar
  - Sometimes allows more natural definitions
  - but, we need disambiguation mechanisms