CSC 425 - Principles of Compiler Design I

Implementation of Lexical Analysis

Outline

- Specify lexical structure using regular expressions
- Finite automata
  - Deterministic Finite Automata (DFA)
  - Non-deterministic Finite Automata (NFA)
- Implementation of regular expressions
  - Regular expression $\rightarrow$ NFA $\rightarrow$ DFA $\rightarrow$ Tables

Regular Expressions in Lexical Specification

- A regular expression specifies a predicate $s \in L(R)$, that is, is a string s a member of the language $L(R)$
- Testing for set membership is not enough, we need to partition the input into tokens
- We can adapt regular expressions to meet this goal

Regular Expressions to Lexical Specification

1. Select a set of tokens
   - Integer, Keyword, Identifier, ...
2. Write a regular expression (or rule) for the lexemes of each token
   - Integer $= [0123456789]+$
   - Keyword $= (\mathit{if} \mid \mathit{else} \mid \ldots)$
   - Identifiers: $[A-Za-z]\ (\ [A-Za-z] \mid [0123456789]\ )*$

Regular Expressions to Lexical Specification

3. Construct a regular expression that matches all lexemes for all tokens
   - $R =$ Integer | Keyword | Identifier | ...
   - $R = R_1 \mid R_2 \mid R_3 \mid \ldots$

- If $s \in L(R)$ then s is a lexeme
  - Furthermore $s \in L(R_i)$ for some i
  - This i determines the token that is reported

Regular Expressions to Lexical Specification

4. Let the input be $x_1 \dots x_n$
   - $x_1 \dots x_n$ are characters
   - For $1 \leq i \leq n$ check if $x_1 \dots x_i \in L(R)$
   - If so, it must be that $x_1 \dots x_i \in L(R_j)$ for some j
   - Otherwise, $s \notin L(R)$
5. Remove $x_1 \dots x_i$ from the input and got to the previous step

Options for Handling Whitespace and Comments

1. We could create a token for whitespace or comments
   - Whitespace = (`' '` | `'\n'` | `'\t'`)+
   - Comment = ...
   - An input of " \t\n 42 " is transformed to the token stream Whitespace Integer Whitespace
2. The lexer skips whitespace and comments
   - This is the preferred method because whitespace and comments are irrelevant to the parser (for most languages)
   - The lexer still needs to match a whitespace (or comment) regular expression, but a token is not output

Ambiguities

- How much input is used?
  - What if $x_1 \ldots x_i \in L(R)$ and $x_1 \ldots x_k \in L(R)$?
  - Rule: choose the longest possible substring ("maximal munch")
- Which token is used?
  - What if $x_1 \ldots x_i \in L(R_j)$ and $x_1 \ldots x_i \in L(R_k)$?
  - Rule: choose the rule listed first (j if $j < k$)

Error Handling

- What if no regular expression matches a prefix of the input?
- Problem: the algorithm needs to terminate
- Solution: write a rule matching all invalid strings and place it at the end of the rules
- Lexer tools allow you to write
  $R = R_1 \mid \ldots \mid \texttt{Error}$
  where the token `Error` matches if nothing else matches

Summary

- Regular expressions provide a concise notation for string patterns
- Adapting regular expressions to lexical analysis requires small extensions to resolve ambiguities and handle errors
- Good algorithms are known that
  - Require only a single pass over the input
  - Require few operations per character (table lookup)

Regular Languages and Finite Automata

- Result from formal language theory: regular expressions and finite automata both define the class of regular languages
- Thus, lexical analysis uses:
  - Regular expressions for specification
  - Finite automata for implementation (automatic generation of lexical analyzers)

Finite Automata

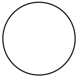
- A finite automata is a *recognizer* for the set of strings of a regular language
- A finite automaton consists of:
  - A finite input alphabet Σ
  - A set of states S
  - A start state n
  - A set of accepting states $F \subseteq S$
  - A set of transitions in $S \to S$ (mappings from states to states)

Finite Automata

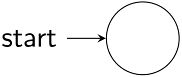
- Transition notation
  $s_1 \ \rightarrow^{a} s_2$
  is read: in state s_1 on input a go to state s_2
- Each transition "consumes" a character from the input
- At the end of input (or no transition possible)
  - If in accepting state, accept ($s \in L(R)$)
  - Otherwise, reject ($s \notin L(R)$)

Finite Automata State Graphs

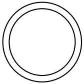
- A state:



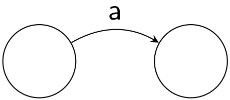
- A start state:



- An accepting state:

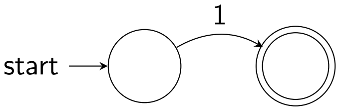


- A transition:



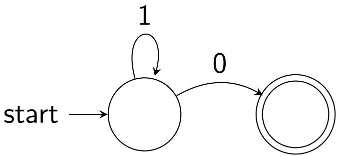
A Simple Example

- A finite automaton that accepts only “1”;



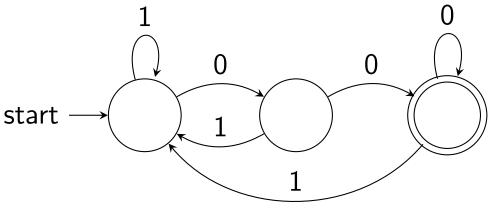
Another Simple Example

- A finite automaton that accepting any number of 1s followed by a single 0
- Alphabet: $\{0, 1\}$



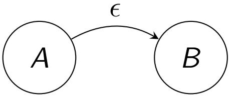
Another Example

- Alphabet: $\{0, 1\}$



Epsilon Transitions

- Epsilon transitions:



- The automaton can move from state A to state B without consuming input

Deterministic and Non-Deterministic Automata

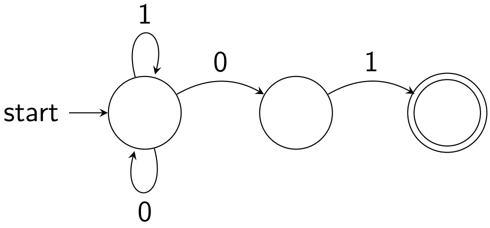
- Deterministic Finite Automata (DFA)
  - One transition per input per state
  - No epsilon transitions
- Non-deterministic Finite Automata (NFA)
  - Can have multiple transitions for one input in a given state
  - Can have epsilon transitions
- Finite automata have finite memory – only enough to encode the current state

Execution of Finite Automata

- A DFA can take only one path through the state graph
  - Completely determined by input
- NFAs can choose:
  - whether to make epsilon transitions
  - which of multiple transitions for a single input to take

Acceptance of NFAs

- An NFA can get into multiple states



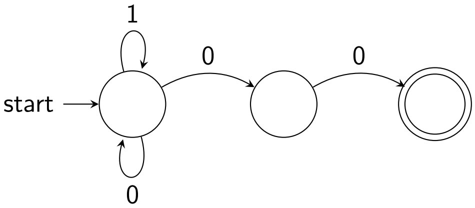
- An NFA accepts an input if it can get in a final state
- Exampe input: 1 0 1

NFA versus DFA

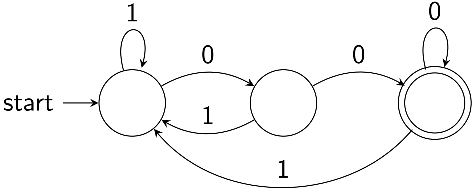
- NFAs and DFAs recognize the same set of languages (regular languages)
- DFAs are easier to implement
- A DFA can be exponentially larger than an equivalent NFA

NFA versus DFA

- For a given language the NFA can be simpler than the DFA
  - NFA:



  - DFA:

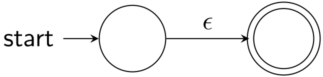


Regular Expressions to Finite Automata

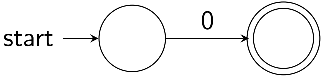
- The implementation of a lexical specification as a finite automata has the following transformations:
    1. Lexical specification
    2. Regular expressions
    3. NFA
    4. DFA
    5. Table driven implementation of DFA

Regular Expressions to NFA

- We can define an NFA for each basic regular expression and than connect the NFAs together based on the operators
- Basic regular expressions
  - ϵ transition

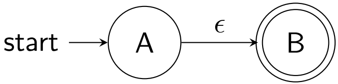


  - Input charater '0'

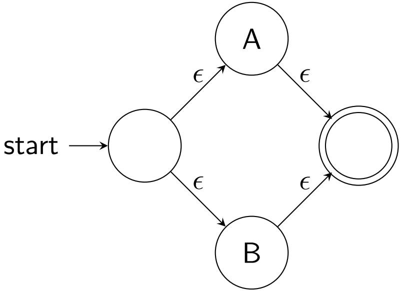


Regular Expressions to NFA

- AB: make an ϵ transition from the accepting state of A to start state of B

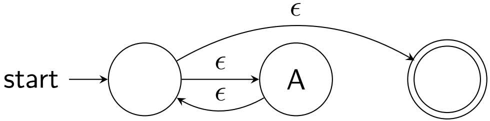


- $A|B$: create a new start state and add ϵ transitions from the new start state to the start states of A and B, then create a new accepting state and add ϵ transitions from the accepting states of A and B to the new accepting state



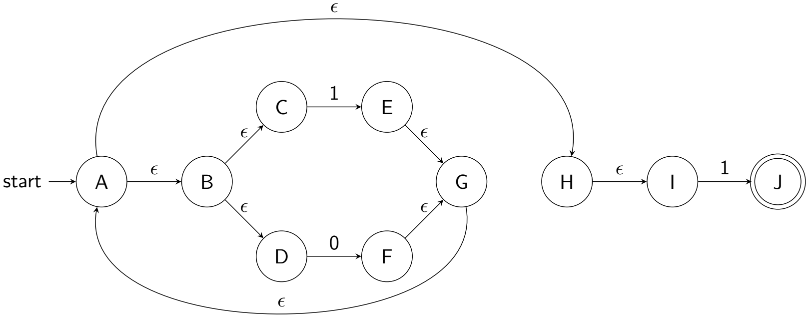
Regular Expressions to NFA

- $A*$: create a new start state and accepting state and add an ϵ transitions: from the new start state to the start state of A, from the accepting state of A to the new start state, and from the new start state to the new accepting state.



Regular Expressions to NFA Example

- Consider the regular expression: `(1|0)*1`
- The NFA is



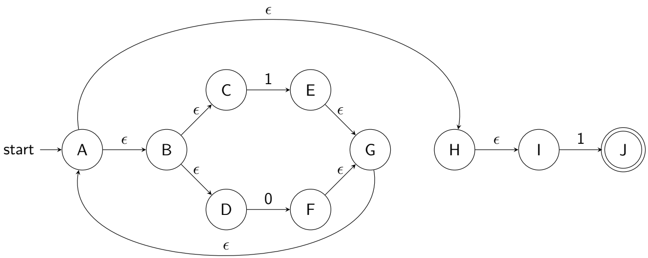
NFA to DFA (The Trick)

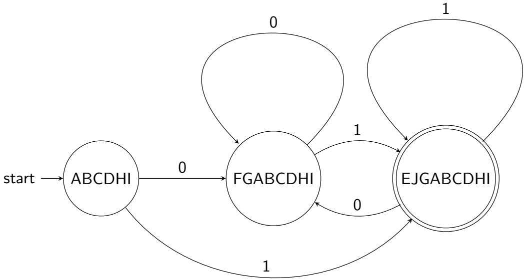
- Simulate the NFA
- Each state of the DFA is a non-empty subset of states of the NFA
- The start state is the set of NFA states reachable through epsilon transitions from the NFA start state
- Add a transition $S \rightarrow^a S'$ to the DFA if and only if S' is the set of NFA states reachable from any state in S after seeing the input a (considering epsilon transitions as well)

NFA to DFA Remark

- An NFA may be in many states at any time
- If there are N states, the NFA must be in some subset of those N states
- There are $2^N - 1$ possible subsets (finitely many)

NFA to DFA Example

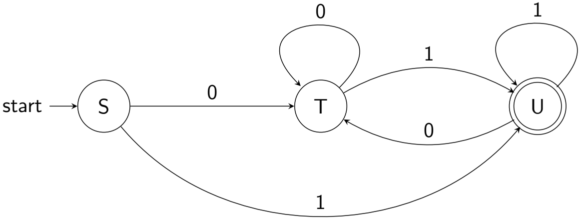




Implementation

- A DFA can be implemented by a 2D table T
  - One dimension is "states"
  - The other dimension is "input symbols"
  - For every transition $S_i \to^a S_k$ define $T[i, a] = k$
- DFA "execution"
  - If in state S_i and input a, then read $T[i, a]k$ and skip to state S_k
  - This is efficient

Example: Table Implementation of a DFA



	0	1
S	T	U
T	T	U
U	T	U

Implementation Continued

- The NFA to DFA conversion is the core operation of lexical analysis tools such as lex
- But, DFAs can be huge
- In practice, lex-like tools trade off speed for space in the choice of NFA and DFA representations

Theory versus Practice

- DFAs *recognize* lexemes. A lexer must return a *type of acceptance* (token type) rather than simply an accept/reject indication
- DFAs consume the complete string and accept or reject it. A lexer must *find* the end of the lexeme in the input stream and then find the *next* one, etc.